Quantization Algorithm

What are the possible results of measurement?

Statistical Algorithm

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i.e. How to evaluate

$$\mathrm{Prob}\left(q \right)_{Q}^{\psi}$$

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$Q \leftarrow$ observable

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Observable is represented by a differential operator

Ex Momentum $\longrightarrow -i\hbar \frac{\partial}{\partial x}$

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i^{th} eigenfunction $\quad$ i^{th} eigenvalue

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then

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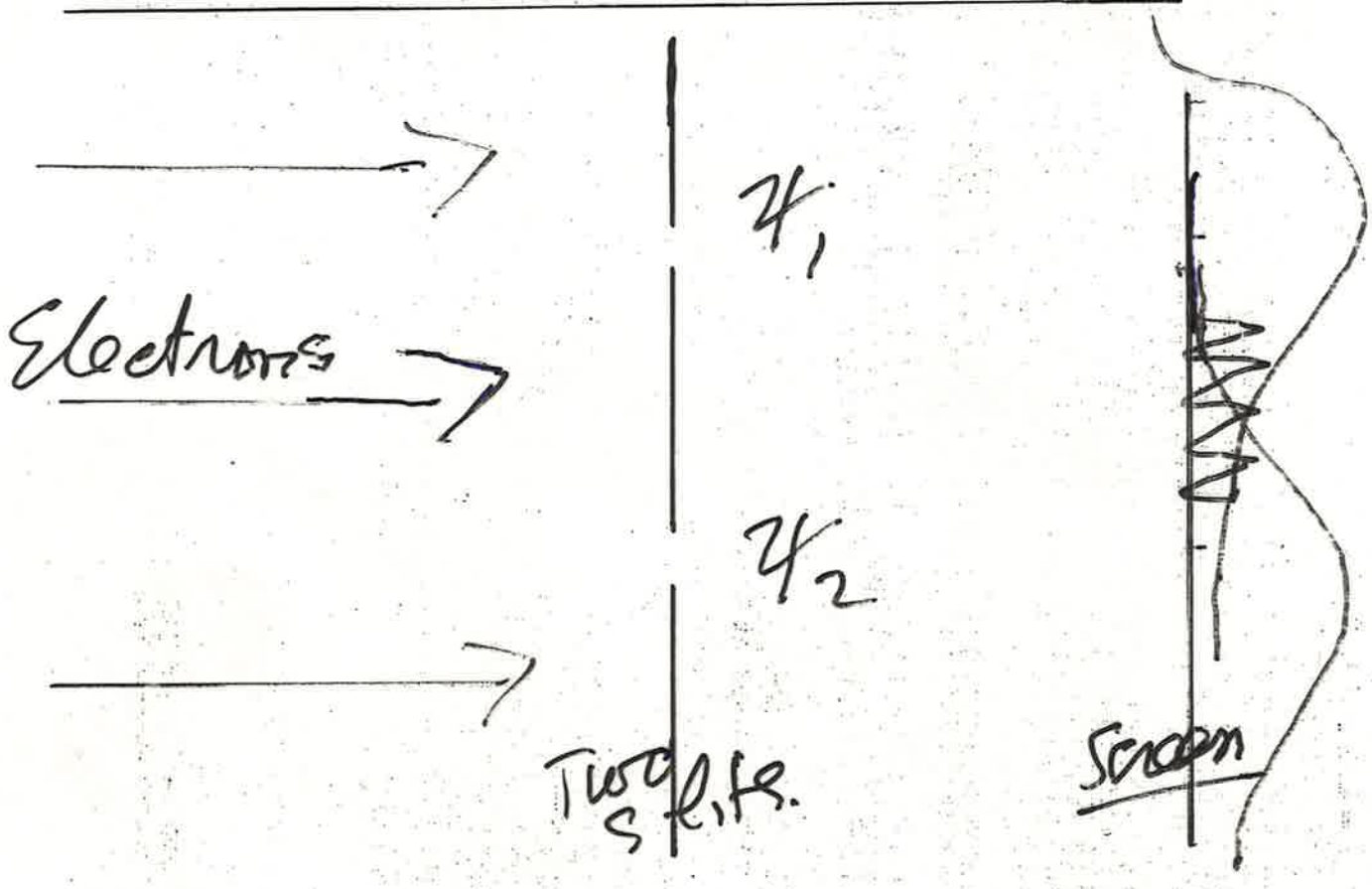
(2) Note that $|\psi_1 + \psi_2|^2 \neq |\psi_1|^2 + |\psi_2|^2$

This is quantum-mechanical explanation of the 2-slit experiment.

TWO-SLIT EXPERIMENT



$$|\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2$$
$$+ 2\,\mathrm{Re}\,\psi_1 \psi_2^*$$

↑ explains the interference pattern

So electron behaves sometimes like a particle and sometimes like a wave — WAVE-PARTICLE DUALITY

Uncertainty Relations

$$\Delta x \, \Delta p \geq \frac{1}{2}\hbar \qquad \left(\hbar = \frac{h}{2\pi}\right)$$

where $\Delta x = \sqrt{\overline{(x-\bar{x})^2}}$

$\Delta p = \sqrt{\overline{(p-\bar{p})^2}}$

cf. γ-ray microscope

$\Delta x \sim \frac{\lambda}{\sin\varepsilon}$

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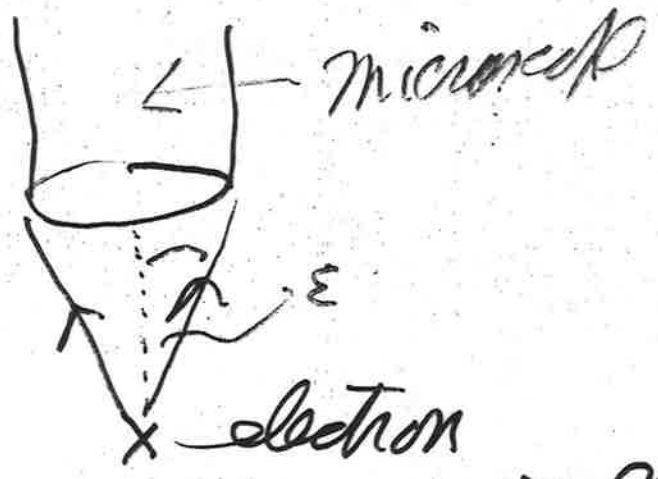

The higher the resolving power, the shorter the wavelength the greater the 'kick' to the electron

View A

QM not mysterious
— glorified Statistical Mechanics

View B

QM is mysterious
— new concepts like potentiality or latency

View C

QM is mysterious, not because of new concepts, but because of limitations on the applicability of old concepts

→ COMPLEMENTARITY

Complementarity

1. grounds for definability are, in general mutually exclusive, experimental arrangements for displaying quantum phenomena.

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→ The moving cut.

observed	observer

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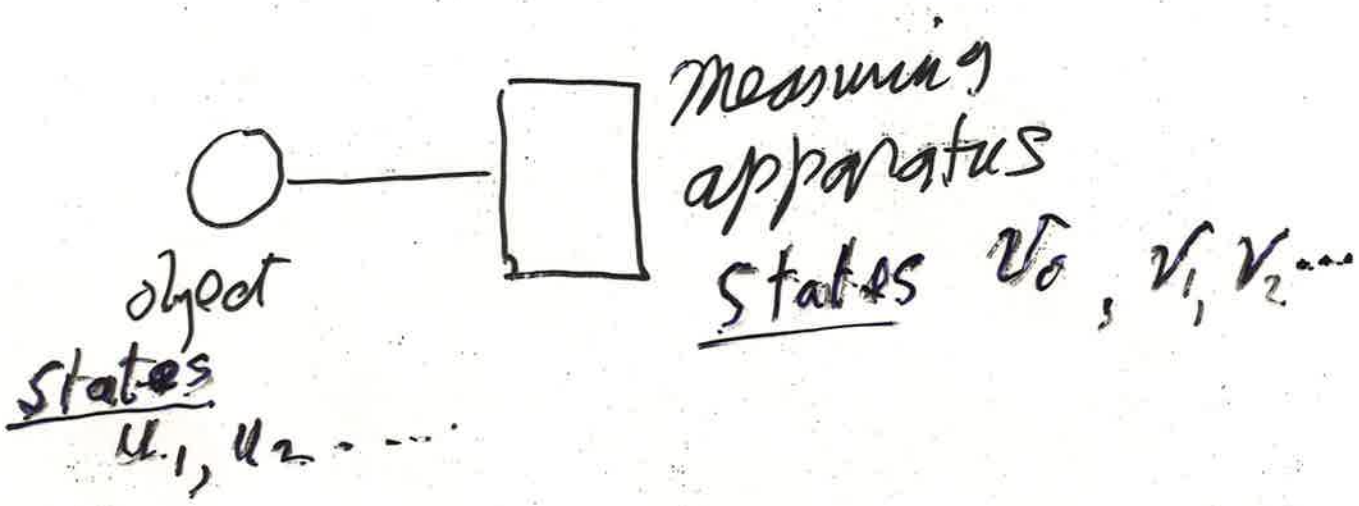
MEASUREMENT IN QUANTUM MECHANICS



$$u_1 v_0 \longrightarrow u_1 v_1$$

$$u_2 v_0 \longrightarrow u_2 v_2$$

Then

$$(u_1 + u_2) v_0 \longrightarrow u_1 v_1 + u_2 v_2$$

$$\Big\downarrow \text{Projection postulate.}$$

$$u_1 v_1$$

$$\text{OR } u_2 v_2$$

Matter

skin epidermis, cortex
phloem organisms, } Ostwald

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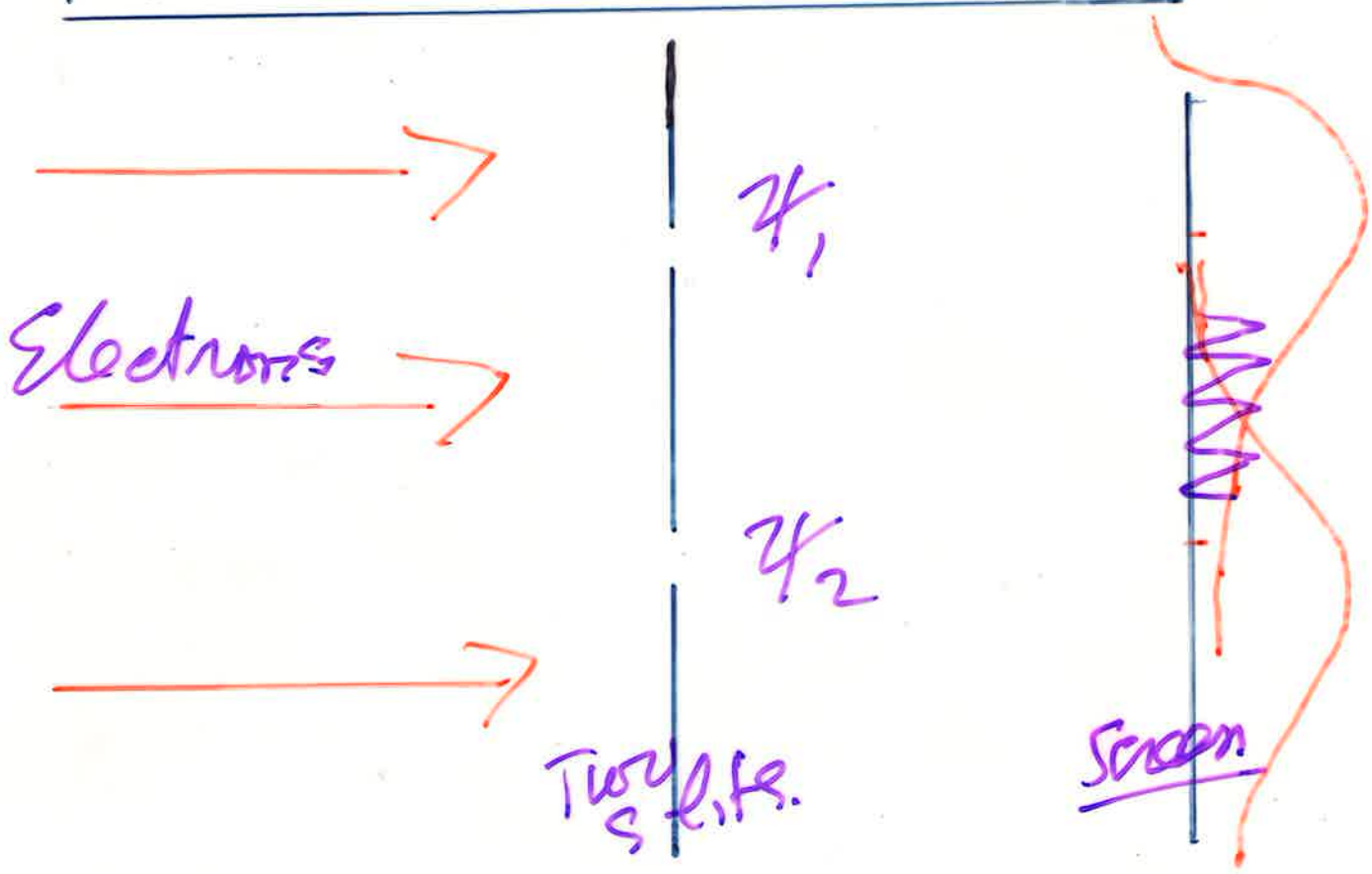

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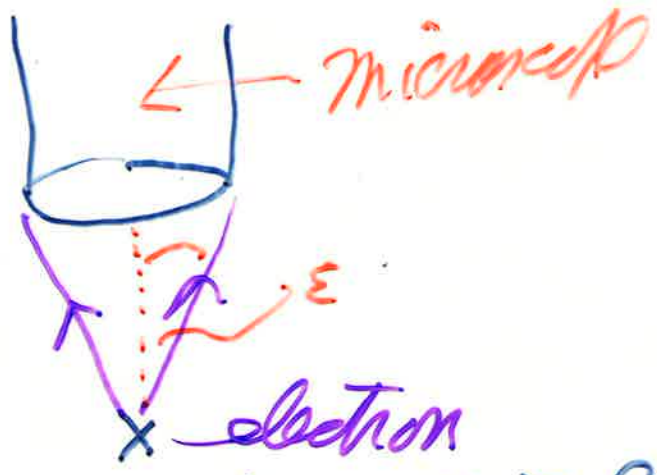

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Additional Transparencies

1 Uncertainties & Statistical Ensembles

2 Uncl - redundancies

3 " " " cont'd

4 2-slit expt.

5 Important relations

6. Views A, B, C. — what they mean

7. Complementarity

8 Measurement in QM

9. Angular momentum & Spin

10 EPR — Reality, Completeness, Locality.

11 Alternative ways of writing Hilbert

12 Stochastic h-v theories
parameter independence
outcome independence.

13. Stoch. theories cont'd — Strict condition
⇒ determinism

(13a) Von Neumann h-v theorem

(14) FVN C.

15. Corr. and Reality Principle - (B)

ANGULAR MOMENTUM AND SPIN

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

Eigenvalues of J^2 are $j(j+1)\hbar^2$

$j = 0, \frac{1}{2}, 1, 3/2 \ldots$

Eigenvalues of J_z are $m\hbar$

$m = -j, -j+1 \ldots j-1, j$

Ex spin $\frac{1}{2}$ i.e. $j = 1/2$.

Eigenvalues of J_z are $\pm \frac{1}{2}\hbar$

Similarly eigenvalues of J_x, J_y are $\pm \frac{1}{2}\hbar$. Define $\underset{\sim}{\sigma} = \frac{2}{\hbar}\underset{\sim}{J}$

Then eigenvalues of σ_z are ± 1, etc.

Eigenstates of σ_z are α, β

where $\sigma_z \alpha = \alpha$, $\sigma_z \beta = -\beta$

Necessary condition for Completeness

Every element of physical reality must have a counterpart in the theory

Sufficient condition for Reality

If we can predict with certainty (or at any rate with probability one) the result of a measurement at time t, then at the time t there exists an element of reality which is revealed by the measurement

Locality Principle

Elements of reality pertaining to one system cannot be affected by measurements performed 'at a distance' on another system.

Note, we can also write

$$\Psi_{\text{singlet}} = \frac{1}{\sqrt{2}}\left(\gamma(1)\cdot\delta(2) - \delta(1)\cdot\gamma(2)\right)$$

where γ and δ are eigenstates of σ_x, with eigenvalues ± 1

i.e. $\sigma_x \gamma = \gamma$

$\sigma_x \delta = -\delta$

It can be shown that

$$\gamma = \frac{1}{\sqrt{2}}(\beta + \alpha)$$

$$\delta = \frac{1}{\sqrt{2}}(\beta - \alpha)$$

HIDDEN - VARIABLE RECONSTRUCTIONS OF QM

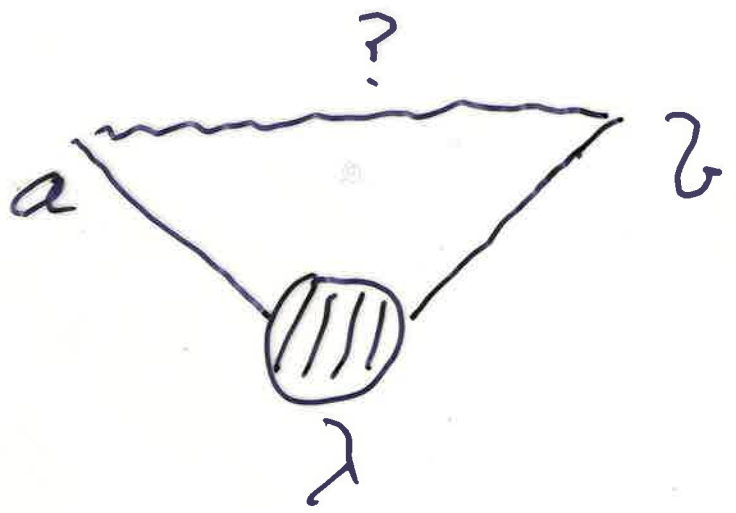
Deterministic

$$\langle Q \rangle_\psi = \int_\Lambda [Q]^\psi(\lambda)\, \rho_\psi(\lambda)\, d\lambda$$

Stochastic

$$\langle Q \rangle_\psi = \int_\Lambda \mathrm{Prob}\left([Q]^\psi / \lambda\right) \cdot \rho_\psi(\lambda)\, d\lambda$$

STOCHASTIC H-V. THEORIES



Assume joint distribution

$$\text{Prob}(a, b, \lambda)$$

$$= \text{Prob}(a \mid b \,\&\, \lambda) \times \text{Prob}(b \mid \lambda) \times \text{Prob}(\lambda)$$

To derive B.I. we require 2 locality assumptions

1. <u>Parameter Independence</u>
   - Marginal distributions do not do not depend on 'distant' settings of apparatus
2. <u>Outcome Independence</u>

$$\text{Prob}(a \mid b \,\&\, \lambda) = \text{Prob}(a \mid \lambda)$$

Hence we can write

$$\text{Prob}(a, b, \lambda)$$
$$= \text{Prob}(a|\lambda) \times \text{Prob}(b|\lambda) \times \text{Prob}(\lambda)$$

Note:

(1) stochastic independence is only assumed for a specified λ

(2) strict correlations

$\Rightarrow$ determinism

This result <u>also</u> follows by an EPR-type argument without assuming a stochastic h-v framework.

What we cannot do (pace Stapp) is to give a proof of B.I. in a non-strictly correlated state without <u>assuming</u> determinism.

Von Neumann.
No-hidden-variable Theorem (1932)

Assumes effectively

* $[A+B] = [A] + [B]$

where A & B do not commute

But this is unreasonable: eigenvalues of (A+B) are not the sum of the eigenvalues of A & B separately.

Bell (1966) & Kochen & Specker (1967) gave no-H-V. proofs restricting * to commuting observables.

Consider an observable in quantum mechanics – call it Q.

Then Q is 'represented' as a mathematical operator

Value of Q is denoted by $\{Q\}$

<u>Query:</u> Can we assign a value $\{Q\}$ to every observable Q in a consistent way that respects Locality

More generally they assumed

FUNC

$$[f(0)] = f([0])$$

This is the Functional Composition Principle

Corr

There is a 1:1 correspondence between the set of self-adjoint operators and the set of observables

Reality Principle (R)

If there is an operationally defined number associated with $\hat{Q}$ (i.e. distributed probabilistically according to the statistical Algorithm of QM) then there exists an element of reality associated with that number and measured by it.

Kochen – Specker Paradox with projection operators

Spin 1

$$P_1 = 1 - J_x^2$$
$$P_2 = 1 - J_y^2$$
$$P_3 = 1 - J_z^2$$

$$P_1 + P_2 + P_3 = 1$$

Spin $\frac{1}{2}$

$$P_1 = \frac{1}{2}(1 + \sigma_z)$$
$$P_2 = \frac{1}{2}(1 - \sigma_z)$$

$$P_1 + P_2 = 1$$